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## C.U.SHAH UNIVERSITY

Summer-2015
Subject Code: 5TE02FEM1 Subject Name: Finite Element Methods
Course Name: M.Tech (CAD/CAM) Date: 20/5/2015
Semester: II
Marks: 70
Time: 02:30 TO 05:30

## Instructions:

1) Attempt all Questions in same answer book/Supplementary.
2) Use of Programmable calculator \& any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams \& figures (if necessary) at right places.
5) Assume suitable \& perfect data if needed.

## SECTION-I

Q. $1 \quad$ Attempt all the following questions
(a) Define finite element method.
(b) Enlist three fundamental steps for solving the problem using finite element method.
(c) Enlist three source of error in a finite element solution.
(d) Draw the sketch of tetrahedron element.
(e) Enlist the different forces acting on an elastic body.
(f) Enlist any two conditions to be satisfied by any interpolation function.
(g) Enlist different methods of applying boundary condition.
Q.2(a) Explain about different types of elements used in FEM.
Q.2(b) Explain about quadratic shape function.
Q.2(c) Explain multipoint method for applying boundary condition.
OR
Q.2(a) An axial stepped bar is shown in the following figure. It is subjected to an axial pull of 50 kN . If the material of the bar is uniform and has modulus of elasticity as 200 GPa , determine the displacements and stresses of each of the section using 1-D element.

Q.2(b) Solve the above problem using conventional method and compare the results.
Q.2(c) Explain about adoptive meshing technique.

Q.3(a) As shown in following figure, load $\mathrm{P}=60000 \mathrm{~N}$ is applied. Determine the displacement field and stress in the body. Take $E=20000 \mathrm{~N} / \mathrm{mm}^{2}$.

Q.3(b) An axial load $\mathrm{P}=300000 \mathrm{~N}$ is applied at $20^{\circ} \mathrm{C}$ to the rod as shown in following figure. The temperature is then raised to $60^{\circ} \mathrm{C}$. Determine nodal displacements and element stresses.


| Element 1 | Element 2 |
| :--- | :--- |
| Aluminum | Steel |
| $\mathrm{L}_{1}=200 \mathrm{~m}$ | $\mathrm{~L}_{2}=300 \mathrm{~mm}$ |
| $\mathrm{E}_{1}=70 \times 10^{9} \mathrm{~Pa}$ | $\mathrm{E}_{2}=200 \times 10^{9} \mathrm{~Pa}$ |
| $\mathrm{~A}_{1}=900 \mathrm{~mm}^{2}$ | $\mathrm{~A}_{2}=1200 \mathrm{~mm}^{2}$ |
| $\alpha_{1}=23 \times 10^{-6}$ per OC | $\alpha_{2}=11.7 \times 10^{-6}$ per OC |

Q.3(c) Evaluate the shape functions $N_{1}, N_{2}$ and $N_{3}$ at the interior point $P$ for the triangular element shown in following figure.


OR
Q.3(a) A two member truss is shown in following figure. The cross sectional area of each member is $200 \mathrm{~mm}^{2}$ and modulus of elasticity is 200 GPa . Determine the deflections in each of the member.




10 KN
Q.3(b) In above problem if increase in temperatures are $30^{\circ} \mathrm{C}$ and $50^{\circ} \mathrm{C}$ for element 1 and 2 respectively, determine deflections in each of the member. Take $\alpha=12 \times 10^{-6}$ per OC.
Q.3(c) Determine the shape functions $\mathrm{N}_{1}, \mathrm{~N}_{2}$ and $\mathrm{N}_{3}$ at the interior point P for the triangular element shown in following figure.


## SECTION-II

Q. 4 Attempt all the following questions
(a) Differentiate between FDM and FEM in terms of lumping of field variable.
(b) Enlist two generalized methods used to choose weighting function in weighted residual technique.
(c) Enlist any two rules for node numbering after descritization.
(d) Enlist criteria to consider 3 D problem as axis symmetrical problem.
(e) Enlist the software used for finite element analysis.
(f) Enlist different methods used for evaluation of eigen value - eigen vector.
(g) Enlist the characteristics associated with non-linear problems.
Q.5(a) A CST element has the nodal coordinates $(15,-8),(10,5)$ and $(2,0) \mathrm{mm}$ for $\mathrm{i}, \mathrm{j}$ and k nodes respectively. The element is 2 mm thick and is of a material having $\mathrm{E}=70 \mathrm{GPa}$ and Poisson's ratio 0.3. Upon loading of the model, the nodal deflections were found to be:


| ui $=100 \mu \mathrm{~m}$ | $\mathrm{uj}=75 \mu \mathrm{~m}$ | uk $=80 \mu \mathrm{~m}$ |
| :--- | :--- | :--- |
| vi $=-50 \mu \mathrm{~m}$ | $\mathrm{vj}=-40 \mu \mathrm{~m}$ | vk $=-45 \mu \mathrm{~m}$ |

Determine Jacobian for $(x, y)-(\zeta, \eta)$ transformation and the straindisplacement relation matrix.
Q.5(b) Consider axial vibration of the steel bar as shown in following figure.

Derive the equation of lowest natural frequency.

Q.5(c) Enlist the various application of FEM.

OR
Q.5(a) Determine the temperature distribution in a composite wall made up of two materials A and B. The length and thermal conductivity of material A are 2 cm and $0.2 \mathrm{~W} / \mathrm{mK}$ and the same of material $B$ are 6 cm and 0.06 $\mathrm{W} / \mathrm{mK}$. The fluid passing over the material A is having temperature of $5^{\circ} \mathrm{C}$ and heat transfer coefficient of $0.1 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$. The outer surface temperature of material B is $20^{\circ} \mathrm{C}$.
Q.5(b) Determine the displacements of nodes of a spring system as shown in figure below using potential energy formulation.

Q.5(c) A vertical shaft is subjected to self weight only. Determine the displacement of 1 m . long steel shaft using 2 element method. Take $\mathrm{E}=$ $2 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}$, density $=7800 \mathrm{~kg} / \mathrm{m}^{3}$ and cross sectional area $=5 \mathrm{~cm}^{2}$.
Q.6(a) A composite wall consists of three layers of aluminum, copper and steel.

The surface temperatures of aluminum and steel are 264 K and 311 K .
Using three element model, determine interfacial temperature and their

heat flow rate per unit area.

| Aluminum | Length $=5.0 \mathrm{~cm}$ | $\mathrm{k}=202 \mathrm{~W} / \mathrm{mK}$ |
| :--- | :--- | :--- |
| Copper | Length $=10 \mathrm{~cm}$ | $\mathrm{k}=388 \mathrm{~W} / \mathrm{mK}$ |
| Steel | Length $=2.5 \mathrm{~cm}$ | $\mathrm{k}=45 \mathrm{~W} / \mathrm{mK}$ |

Q.6(b) A problem of 2-D heat conduction problem domain is shown in
following figure. Evaluate the stiffness matrix for conduction and convection for element number 1.


$$
\mathrm{h}=2 \mathrm{~W} / \mathrm{cm}^{2 \mathrm{O}} \mathrm{C}, \mathrm{~T} \alpha=30^{\circ} \mathrm{C}
$$

$\mathrm{kx}=\mathrm{ky}=2 \mathrm{~W} / \mathrm{cm}-{ }^{\circ} \mathrm{C}$, thickness $=1 \mathrm{~cm}$
Q.6(c) Differentiate between Numerical Methods, FDM and FEM.

## OR

Q.6(a) A pin fin is attached to the base plate having temperature of $100^{\circ} \mathrm{C}$. The diameter and thermal conductivity of pin fin are 1 cm and $3 \mathrm{~W} / \mathrm{cm}-\mathrm{K}$. The environmental temperature and heat transfer coefficient are $10^{\circ} \mathrm{C}$ and $0.2 \mathrm{~W} / \mathrm{cm}^{2}-\mathrm{K}$. Using one dimension two elements, calculate the temperature distribution in the pin fin. Take length of fin as 1.5 cm . Consider the case of finite length of fin losing heat from the tip.
Q.6(b) Solve the above problem using conventional method and compare the results.
Q.6(c) Enlist the properties of stiffness matrix.


